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Between the sound of static and water boiling.

For more than a thousand years the "science and the art of tones" called music has been composed using mathematic intervals (Webster). Music's popular definition includes rhythm: the periodic recurrence of accent with symmetry of movement and accent (Webster). Access to music has increased dramatically within the recent past thanks to the Internet, so composers have to push harder than their predecessors to produce unique compositions. The study of math as a basis for music was thoroughly explored in the 1940's when Joseph Schillinger published more than a dozen books on music composition. His research introduced algorithmic composition, a groundbreaking idea that by following a simple formula one could generate rhythm in music. Now, with the use of computers, composers are using chaos theory to enhance algorithmic music composition. Chaos is seemingly unpredictable behavior due to high sensitivity of initial conditions. Weather is considered chaotic, for example, a butterfly's sneeze affects the weather enough to cause a hurricane weeks later. Rhythm in music can be adapted to chaos theory using fractals. A fractal is a shape independent of scale (scaling invariance), and self-similar meaning it can be subdivided into parts, each of which is a smaller copy of the whole (Taylor). Fractals can form basic rhythm using self-similarity and scaling invariance. 18th century composers like Bach and Mozart used these ideas for their compositions, two centuries before Benoit Mandlebrot coined the term "fractal." Using chaos theory for algorithmic composition one can create an endless variety of rhythmic patterns for music.

Patrick Swickard, in his report titled Fractals, Chaos, and Music covers discoveries between fractals and music, controversies over conflicting discoveries, relations between mathematics and music, and Joseph Schillinger's work on music. Swickard provides some insight into the history of mathematics and music starting during the 5th century BC. The Pythagoreans in Greece first provided scientific means of expressing musical intervals as numeric proportions (Swickard). More recently, Joseph Schillinger, a music theorist, did an intense mathematical study of music in between the 1920's and 1930's. Schillinger offers a synopsis of the three fundamental periods in the history of musical instruments: a mammal or man uses the organs of his body, the man starts to use objects of the surrounding world (shells, horns) and then the discovery of scientific methods of sound production (11). His example describes a scene that sound started as a spontaneous reflex of the vocal chords induced by fear that then became a signal for approaching danger (13). Schillinger's ideas on rhythmic design are from the modern tendency in science that all physical phenomena are derivatives of the properties of space, where time is one of the components. Therefore art or rhythm in this case, can be measured and analyzed since it's a derivative of the space-time continuum: "a design is rhythmic if analysis reveals the regularity in the sequence of its components and their correlations" (365).

Shortly before Schillinger's death in 1943 he published a twelve-book collection titled "The Schillinger System of Musical Composition," an example of his passion for developing a scientific way to approach music with mathematics. His writings laid the foundation for algorithmic composition, a technique not expanded on for decades. Swickard's report examines the relationship between fractals and the Schillinger System of Musical Composition using the following ideas of Schillinger: "There are two sides to the problem of melody: one deals with the sound wave itself and its physical components and with physiological reactions to it. The other deals with the structure of melody as a whole, and esthetic reactions to it. Further analysis will show that this dualism is an illusion and is due to considerable quantitative differences. The shore-line of North America, for example, may be measured in astronomical, or in topographical, or in microscopic values." (Swickard) Swickard points out that the same argument was used years later by the founder of fractal geometry, Benoit Mandelbrot, when describing the fractal nature of a coastline and how the length seems to change depending on how finely it is measured (Swickard).

Published after Schillinger's death, The Mathematical Basis of the Arts is his explanation of the mechanism of creatorship as it manifests itself in nature and in the arts. Schillinger's laws of rhythm, are described as general esthetic laws that are based on two fundamental processes: the generation of harmonic groups through interference, and the variation of harmonic groups through combinatory and involutionary techniques (4). He presents the idea that the art of making music consists on arranging the motion of sounds (pitch, volume, quality) so they appear to "organic, alive" (5). Schillinger describes the general method of producing rhythmic sequence is based on the physical phenomenon known as interference. For example, a sound wave with a length or period of three could be combined with another wave with a period of four. The combined attacks of both waves, when divided by the least common denominator, would form a new rhythm with notes of length three, one, two, two, one, three. Swickard describes this by stating “simple rhythms could be found by sort of superimposing two waves of different periodicities and forming a new wave that contained the attacks of both waves." (Appendix, Figure 1) Swickard also points out the Schillinger method to produce rhythmical patterns using distributive powers. One example of distributed involution grouping consists of taking a series of fractions that added up to 1 in the form (a+b), and squaring it to come up with a new pattern (Swickard). Schillinger's idea of rhythm fits the form a fractal would take.

During the 1970’s Richard Voss and John Clarke devised an even more general mathematical study of music, the physical sound of the audio as played, rather than the written structure. They used spectral density, the quality measured by monitoring the voltage used to drive the speakers through which the audio signal is played. Spectral density is used to analyze random signals or noise and is used to determine the behavior of a quantity varying over time. The autocorrelation function was also used to measure how the fluctuations in the signal related to previous fluctuations. Swickard points out that the concepts of spectral density and autocorrelation have been explained by Mandlebrot in the following manner: If one takes a tape recorder, he argues, and records a sound, then plays it faster or slower than normal, the character of the sound often changes considerably. Some sounds, however, will sound the same as before if they are played at a different speed; one only has to adjust the volume to make it sound the same. These sounds are called "scaling sounds." Swickard provides examples of two different types of scaling sounds, white noise and Brownian noise:

> The simplest example of a scaling sound is white noise, which is commonly encountered as static on a radio. This is caused by the thermal noise produced by random motions of electrons through an electrical resistance. The autocorrelation function of white noise is zero, since the fluctuations at one moment are unrelated to previous fluctuations. If white noise is recorded and played back at a different speed, it sounds pretty much the same: like a "colorless" hiss. In terms of spectral density, white noise has a spectral density of 1/f^0.
>
> Another type of scaling sound is sometimes called Brownian noise because it is characteristic of Brownian motion, the random motion of small particles suspended in a liquid and set into motion by the thermal agitation of molecules. Brownian motion resembles a random walk in three dimensions. Since where a particle goes next does depend on its current position, Brownian motion is random but still highly correlated.

> Similarly, Brownian noise is much more correlated than white noise, since the fluctuations at a point in time do depend on previous fluctuations and cannot stray too far from them in too short a time. Brownian noise has a spectral density of 1/f^2. ()

Voss and Clarke's investigation of scaling sounds determined the spectral density of the audio signal did not provide the results they were interested in, instead they decided to monitor the power provided to the speakers rather than the voltage. With this new method, the team discovered the audio power exhibited 1/f behavior, between white and Browning noise. Swickard presents the idea that 1/f spectral density is also present in other phenomena such as electronic flicker noise, sunspot activity, uncertainties in time as measured by an atomic clock, the wobbling of the Earth’s axis, traffic flow on freeways, and even the flood levels of the river Nile. This spectral density behavior of 1/f also matches different kinds of music such as Bach’s First Brandenburg Concerto and piano rags of Scott Joplin, even different radio stations demonstrated this behavior: rock, classical, and even a talk station (Swickard).

Once Voss and Clarke found music exhibited 1/f behavior, they decided to compose music using the noises they studied. Using the groundwork laid by Schillinger, Voss and Clarke formulated an algorithmic music composition technique for the application of scaling sounds. Compositions based on white, 1/f, and Brownian noises were played for listeners who commented on the pieces. The listeners noted that the white music seemed too random, and the Brownian noise seemed too correlated. The 1/f music seemed the most like regular music to listeners, so the team of Voss and Clarke took this as more evidence for the 1/f nature of music. Voss then started to experiment with algorithmic composition of 1/f music using natural phenomena as starting points, one composition was derived from annual flood levels of the Nile.

Written in 1924, Lejaren Hiller's Experimental Music characterizes the process of musical composition as involving a series of choices of musical elements from a variety of musical materials and asserts that the act of composing can be thought of as the extraction of order out of a chaotic multitude of available possibilities. Hiller provides insight on musical composition as the extraction of order within chaos with the idea formulated during the 14th century BC by Aristoxenus who stated: "The voice follows a natural law in its motion and does not place the intervals at random"" (16). Hiller also points out that Aristoxenus also recognized the necessity of ordering in music and language: "The order that distinguishes the melodious from the unmelodious resembles that which we find in the collocation of letters in language. For it is not every collocation but only certain collocations of any given letters that will produce a syllable" (17). Hiller quotes contemporary author Igor Stravinsky's Poetics of Music to illustrate defense for the principle of opposing order and design to chaos: "… we feel [the necessity] to bring order out of chaos, to extricate the straight line of our operation from the tangle of possibilities" (17); that "… we have recourse to what we call order … order and discipline" (17). Hiller presents Stravinsky's definition of art as the "… contrary of chaos. It never gives itself up to chaos without immediately finding its living works, its very existence threatened" (17). Hiller explains that the selection of certain materials out of a random environment makes it obvious that all music falls between order and chaos and changes in musical style involve fluctuations towards one extreme or the other (17). Hiller expands on this idea with a passage from Leonard Meyer: "shape may, from this point of view, be regarded as a kind of stylistic 'mean' lying between the extremes of overdifferentiation and primordial homogeneity" (18).

David Little's COMPOSING WITH CHAOS; Applications of a New Science for Music points out concepts and mathematical models from Chaos Science can be applied to composition using examples of his own works. Little starts by discussing the work of Edward Lorenz, a meteorologist during the 1960's, who modeled the Earth's weather on a computer using proven physical laws of gas and water behavior. Lorenz encountered a problem: if he started the simulation with initial conditions that were slightly different, the weather would diverge and end up completely different. Lorenz determined that small errors in measurements would multiply, leading to his theory, the "Butterfly Effect," that a butterfly stirring its wings in Peking could start a storm over New York the next month (Little). Later, Lorenz developed a mathematical model for the behavior of heated fluid also called convection. This equation for the model of convection used three variables in a non-linear relationship (might show bends, reversals, etc., opposed to a linear relationship that shows a straight line.) This new function calculates new values for each variable dependent on its last value. These changing variables can be traced using a phase diagram. Points on this diagram represent the physical state of this system in three dimensions. If a system heads towards a stable state, its phase diagram will localize to a point called the attractor. (Appendix, Figure 2) Little explains that "Lorenz's model appears to be chaotic, with a kind of infinite complexity; it has a strange attractor!" (Little). As it turned out, Lorenz's model loops endlessly without repeating or crossing itself, flipping unpredictably between sides. However, it is not random and remains within certain bounds: a pattern emerges resembling butterfly wings. (Appendix, Figure 3) This was a discovery that is recurrent in many natural phenomena: order within chaos (Little).

Using these ideas, David Little composed Harpsi-kord for tape and harpsichordist is in 1988 with the central idea of order within chaos. This piece was composed swinging between the poles like the butterfly effect, from regular to irregular, loud to soft, atonal to harmonic, and use of timbre from an ancient instrument or electronic sound. The harpsichordist relates to the tape in a somewhat improvisational manner, with timing and pitch notated, but the rhythm and order improvised. Another composition by Little in 1988, Shuffle, uses a computer to compose, produce and manipulate the music using an 88-note chromatic scale. The computer stores each note, controlled by MIDI information that stores pitch, timing/length, and loudness into memory units. The data is then shuffled around by the compute, for example two randomly chosen memory units for pitch were exchanged. Output is used for input as a sort of feedback process, with each cycle becoming more diffuse and irregular. The ordered chromatic scale slowly degenerates into a super-serial shuffled mix, with the final state very complex, dependent on the cumulative effect of many small random choices. As a result, Shuffle is a musical model of the butterfly effect, a composition described in chaos theory as a sensitive dependence to initial conditions. (Little)

Another technique found in chaotic composition uses the mathematical model for population growth, called the logistic difference equation. First derived in 1845 by P.F. Verhulst, the basis for Verhulst's work was the Malthusian Model describing unbounded growth of a population where xnew=a*x. This formula finds the population of a new generation by multiplying the number in the last generation by a productivity factor. For example assume that the population doubles each generation, therefore a=2. Starting with 2 parents, they would have 4 children, 8 grandchildren, etc., and by generation 10 there would be 1024 siblings. Verhulst wanted to make a more realistic model population growth so he assumed that in nature the larger a population gets the less productive it becomes. As a result, his model limited the upper limit of a population to 1, so the room left over for a new generation is 1-x, the correction factor to unbounded growth. Therefore, the Verhulst Model for population growth becomes: xnew=a*x*(1-x). A new generations population is equal to the Malthusian growth factor times the old population scaled down by the amount of room available for growth (Little). When the productivity factor is set at 2 with a low seed value like 0.001, population x rises and levels off at 0.5. This might be expected in nature with animals with healthy productivity (Little). The initial period is rapid growth, eventually stabilizing. Once the productivity factor is set to a higher value such as 3.2, x grows rapidly at first but doesn't stabilize, instead it alternates between two values endlessly, regardless of the seed value. After a is set larger than 3.569946 x starts to fluctuate chaotically from one value to the next, sometimes moving between values only to spin off again. This is another example of chaotic behavior occurring in nature, with an animal with high productivity that outgrows the environment to support itself, the population crashes and builds up again. This model does show some kind of regularity with x-values going up and down but never repeating exactly. While searching for the exact values of a where the behavior of the model changed-where x values would settle down eventually to one, two for or eight values, Mitchell Feigenbaum discovered a constant ratio between the a values (Little). Other mathematical formulas such as the onset of turbulent flow also showed these doublings and the same ratio between them. This discovery is considered a universal constant, like the constant of gravity, the speed of light, or the weight of an electron. Increasing a past 3.83 in the Verhulst model, the chaotic behavior stops and x circles between 3 values. Increasing a past this, chaotic behavior reoccurs until a=4, our upper limit because values of a greater than this would produce x values greater than 1 which exceeds the definition of the maximum population. Using a computer, a graph can show how the Verhulst formula behaves for all settings of the a value. (Appendix, Figure 4) The pattern contains its own replica within itself. (Appendix, Figure 5) This kind of nested pattern is called a fractal, derived from the Latin adjective fractus, meaning irregular or broken by Mandelbrot . Fractals are characterized by intricate patterns within patterns, with self-similarity on any scale. These fractals occur in natural phenomena and shapes such as trees and clouds. Little's composition Fractal Piano 6 uses the Verhulst model, known as the logistic difference equation as inspiration. For this composition, values obtained from iteration of the equation were encoded using non-linear mapping into pitches, lengths, and loudness. This MIDI data was expanded and compressed with regard to time and layered in different ways using fractal structures. For example, an on/off pattern was used to create fragmented density in parts. (Little)

Chaos, Self-Similarity, Musical Phrase and Form, a record of exploration by Gerald Bennett, is an examination of two aspects of chaotic systems: self-similarity and scaling invariance and his reflection on their appropriateness in musical composition. Bennett describes different musical uses for the logistic difference equation, using the envelope to derive pitch or amplitude, to drive a filter or describe the formal evolution of a section or an entire piece. He describes Gary Nelson's composition "The Voyage of the Golah Iota": how the function's envelope determines the form of composition and how the function drives a granular synthesis routine to produce sound.

Gary Nelson describes granular synthesis techniques, logistic maps (chaos), genetic algorithms and other methods of composition used in his own works for his paper Wind, Sand, and Sea Voyages: An Application of Granular Synthesis and Chaos to Musical Composition. Nelson presents granular synthesis as "a large number of small sounds [...] assembled in masses in a manner that parallels the pointillist painters who created images from small colored dots" (Nelson). These frequencies, amplitudes, timbres, and the distribution of grains form larger sound structures. His composition, The Voyage of the Golah Iota uses the logistic difference equation for a source of the notes. Nelson varies the a value from 1.0 to 4.0 then back to 1.0 during the course of the song. The values of x were iterated, scaled and then mapped onto a 7 octave pitch range, taking advantage of the bifurcations when a is increased, leading to and away from chaotic behavior.

It is important to note that the central characteristics of chaotic systems, scaling invariance and self-similarity were used in musical compositions well before the ideas, terms, and definitions were fully developed. Bennett uses Johann Sebastian Bach's Kunst der Fuge from 1749 as an example: "The choral melody, slightly ornamented, is in the upper voice. The other voices prepare for the entry of the choral by imitating the melody in diminution (twice as fast). The alto voice plays the inversion of the melody, and most of the other accompanying material is derived directly from the opening measures. The entire piece consists of three more phrases, all treated in the same way." () In this example, self-similarity consists of the repeated use of the same motives within a larger section of the whole. Bennett provides insight into Bach's late composition, the title: "I herewith stand before Thy throne" and points out Bach signs his name numerologically in the choral melody twice. (Bennett)

Using ideas from Chaos Science, twenty-first century composers will realize the benefits of fractals in music. Electronic music synthesis and its human interaction will evolve beyond the traditional analog instrument. The basis of these innovations are thanks to the work of Joseph Schillinger in the early twentieth century. The thoughts of Hiller and Stravinsky express the need for human intervention during composition of algorithmic music. The experimentation by Voss and Clarke introduced many to a new genre when electronic music can be created using mathematic sound synthesis, even reflecting occurrences in nature. David Little and Gary Nelson exploited fractals for composition in the late twentieth century. As electronic age progresses the need for human intervention will lessen during composition. Ideas of artificial intelligence, cellular automation, granular synthesis, and fractals will merge with electronic synthesis.

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Frequently asked questions for sci.fractals Internet newsgroup.

Appendix

Figure 1 (Swickard)

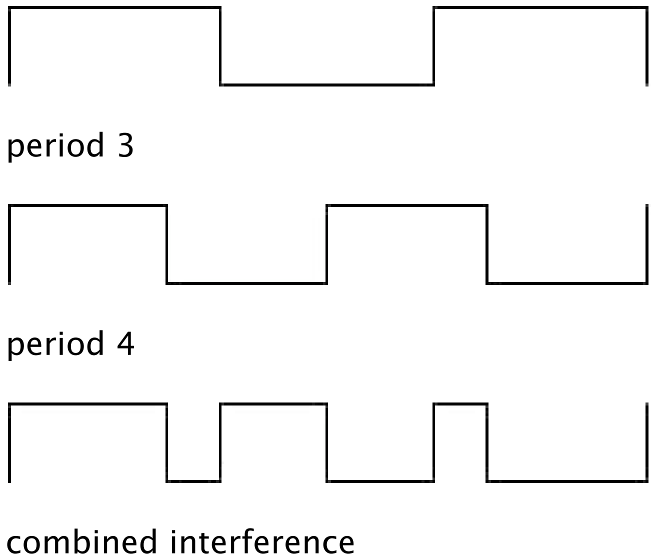
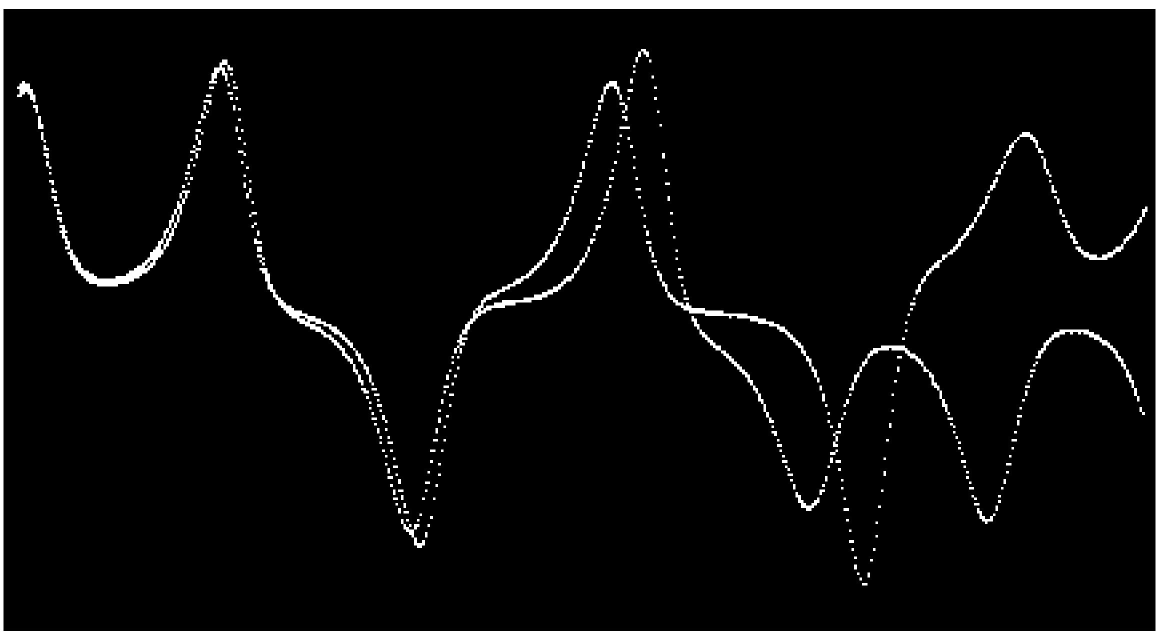
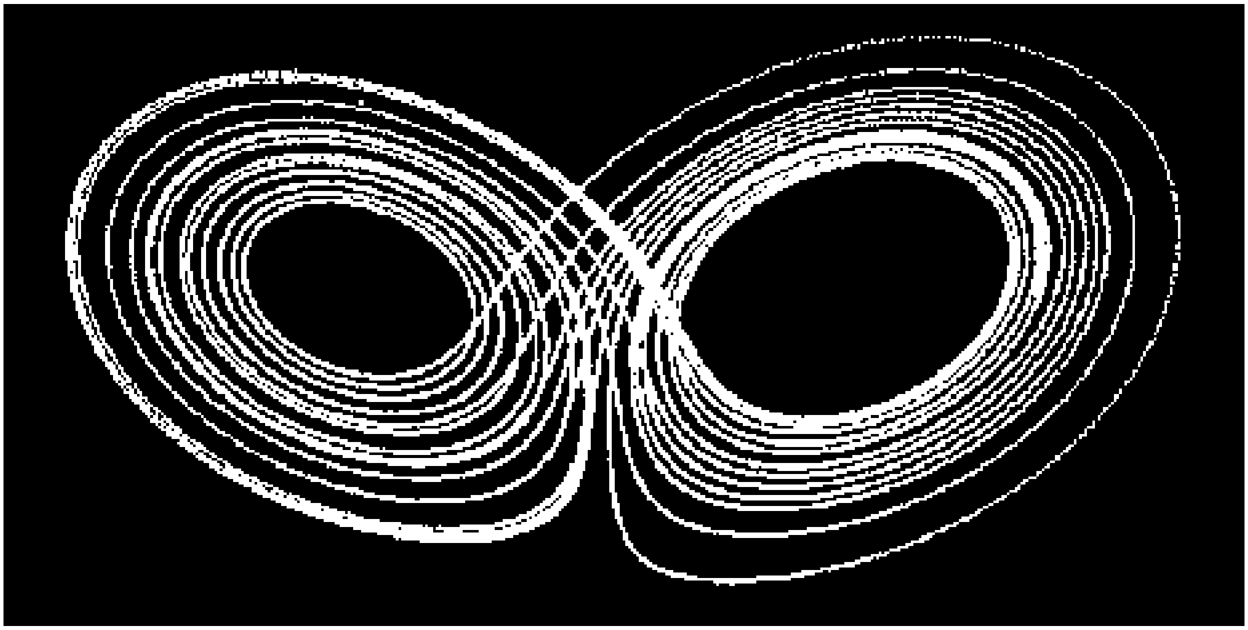


Figure 2 (Little)



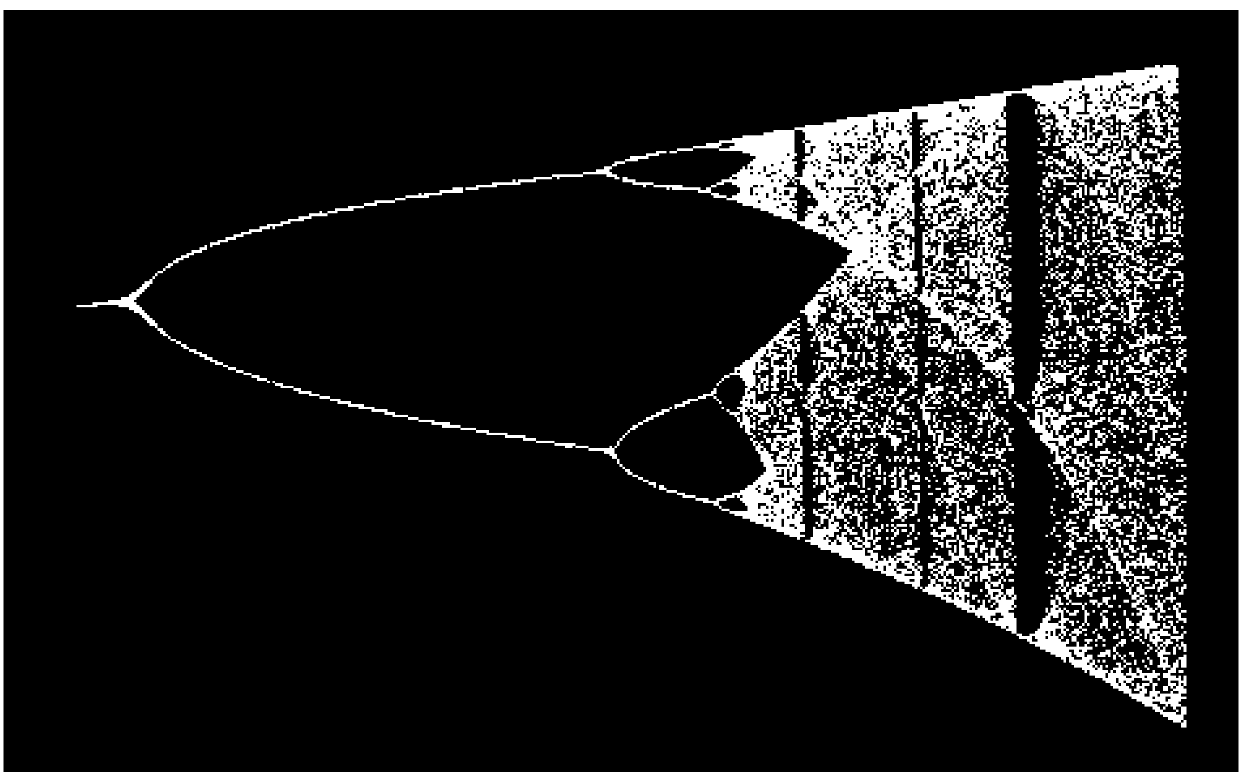
The Butterfly effect

Figure 3 (Little)



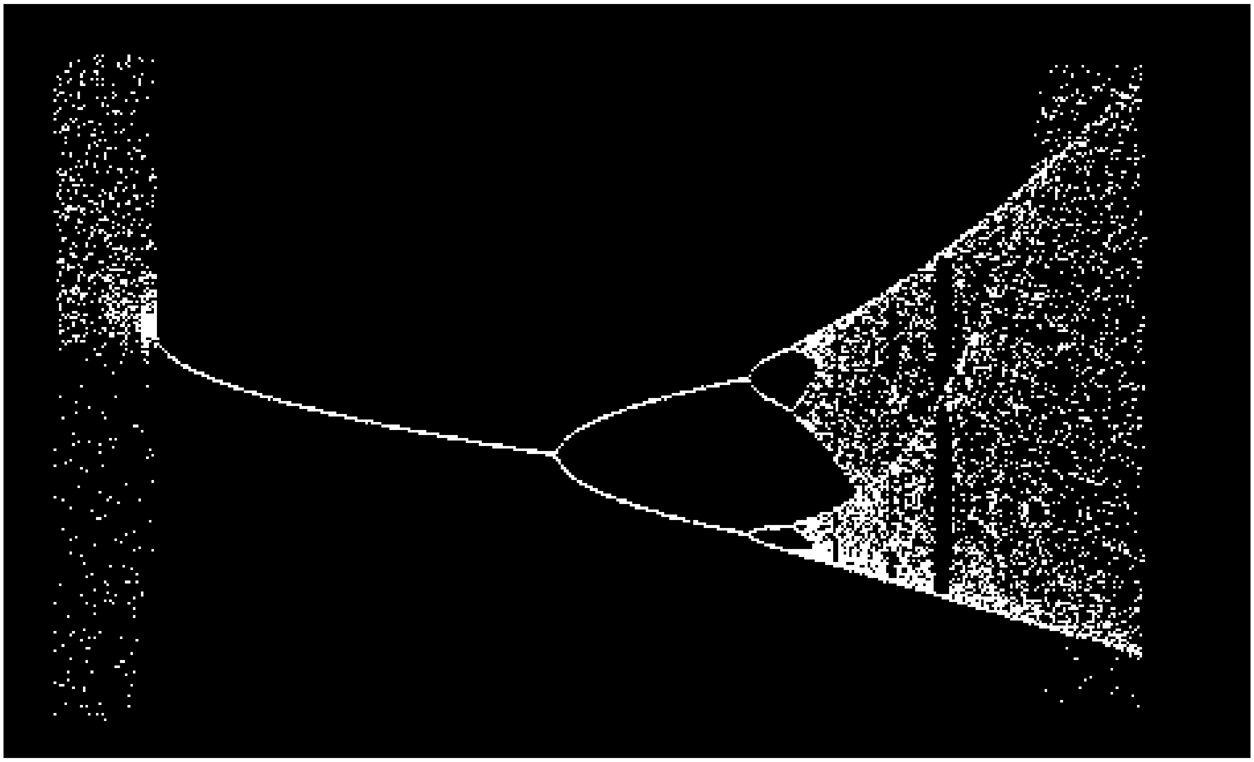
Lorenz Function

Figure 4 (Little)



Verhulst model for population growth.

Figure 5 (Little)



Verhulst model magnified at bifurcation point.